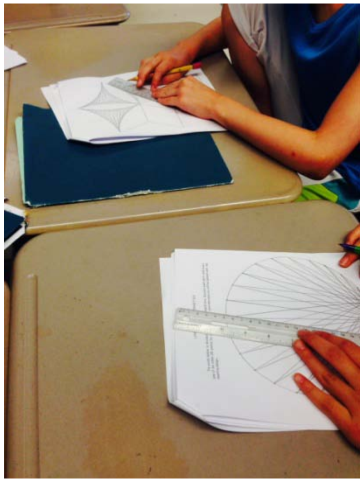
Bedford Public Schools

Grade 4 – Math

The fourth grade curriculum builds on and extends the concepts of number and operations, measurement, data and geometry begun in earlier grades. In the area of number and operations, students generalize place value understanding for multi-digit whole numbers and use this understanding and the properties of operations to perform multi-digit arithmetic. They are expected to have mastered multi-digit addition and subtraction and apply these to solve multi-step problems. They also are expected to master all multiplication and division facts to 100. They extend their understanding of multiplication to multiplying a one-digit number by a four-digit number and by multiplying a two-digit number by a two-digit number and use these skills in problem-solving situations. They also learn to divide up to a four-digit number by a one-digit number. Students continue the work with fractions that they began in third grade. They develop an understanding of fraction equivalence as well as addition and subtraction with fractions and mixed numbers with the same denominators, and multiplying a fraction by a whole number. They extend their understanding of fractions with denominators of 10 and 100 by expressing these as decimals. They then are expected to know how to compare two decimals to the hundredths. In geometry, students deepen their understanding of 2-dimensional shapes by drawing and identifying lines and angles, and classifying the shapes by properties of lines and angles. In measurement, students are introduced to the concepts of angle and angle measurement, are expected to master the concepts of perimeter and area of two-dimensional shapes, and continue their experience with standard and metric units of measure while being expected to know relative sizes of measurement units in one system of units.

Throughout all grades, there is an emphasis on the skills of mathematical practice that prepare children to be mathematically proficient students. These skills include making sense of problems and persevering in solving them, assessing how reasonable their answers are, explaining in words (both orally and in writing) their understanding and reasoning, attending to precision in both calculations and in math language, using appropriate math tools, and looking for and extending patterns.

Assessments happen in multiple ways routinely throughout the school year to measure student progress. Assessments consist of informal as well as formal teacher observations, small group interviews, individual interviews and checkpoints, and written tests. Student progress is monitored carefully to ensure proficiency in both the mathematical content and the practice standards that are expected at each grade level.



Learning Expectations

Operations and Algebraic Thinking

Enduring Understandings In order to meet the standards, the students will need to understand that . . .	**Essential Questions** In order to understand, students will need to consider questions such as . . .	**Knowledge and Skills** Learning this material will require students to . . .
Use the Four Operations With Whole Numbers to Solve Problems • Some real-world problems involving joining, separating equal groups, or comparison can be solved using multiplication; others can be solved using division. • Statements of comparison ("times as many") are other ways to think about multiplication or division. • Multiplication and division have an inverse relationship. The inverse relationship between multiplication and division can be used to solve problems. • There is more than one way to estimate a sum, difference, product or quotient. Each estimation technique gives a way to replace numbers with other numbers that are close and easy to compute with mentally. • Rounding is one way to estimate answers. It involves replacing numbers with the closest multiple (or power, if appropriate) of 10 or 100. • Mathematical situations can be represented with equations using letters for the unknown quantity which may appear in any position. • When dividing the remainder must be less than the divisor. The nature of the question asked determines how to interpret and use the remainder.	• What are different interpretations of multiplication and division? • What models can be used to show problems using multiplicative comparisons? • How can the inverse relationships between addition and subtraction, and multiplication and division be helpful in solving problems? • How can addition, subtraction, multiplication and division situations be represented in equations using a symbol for the unknown? • What are strategies for assessing the reasonableness of answers? • What are ways to represent problems using equations with letters standing for the unknown in any place? • What strategies are most helpful for solving particular problems?	• Interpret a multiplication equation as a comparison. (i.e. Interpret 35 = 5 x 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. • Multiply or divide to solve word problems involving multiplicative comparison. • Solve multi-step word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess for reasonableness using computation and estimation strategies including rounding.

Enduring Understandings In order to meet the standards, the students will need to understand that . . .	**Essential Questions** In order to understand, students will need to consider questions such as . . .	**Knowledge and Skills** Learning this material will require students to . . .
Gain familiarity With Factors and Multiples • Every counting number is divisible by 1 and itself, and some counting numbers are also divisible by other numbers. Divisibility rules are helpful. • Some numbers have exactly two factors, 1 and itself. These are prime numbers. • Some numbers have more than two factors. These are composite numbers. • The product of any nonzero number and any other nonzero number is divisible by each number and called a multiple of each number. • A product is the result of multiplying numbers that are called factors.	• How can patterns and properties be used in finding factor pairs for whole numbers in the range of 1-100? • What is the difference between factors and multiples? • How can one determine whether a number is prime or composite?	• Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. • Determine whether a given whole number in the range 1–100 is prime or composite
Generate and Analyze Patterns • Relationships can be described and generalizations made for mathematical situations that have numbers or objects that repeat in predictable ways. • Some patterns consist of shapes or numbers that are arranged in a unit that repeats. • Some numerical sequences have rules that tell how to generate more numbers in a sequence. • Some sequences of geometric objects change in predictable ways that can be described using mathematical rules. • Some real-world quantities have a mathematical relationship; the value of one quantity can be found if you know the value of another quantity.	• How can relationships between numbers or objects that repeat in predictable ways be described and generalized? • How can patterns be used to describe how quantities or shapes are related? • How can a relationship between two quantities be shown using a table? • How can clear explanations be stated or written that describe the relationships between numbers or objects that repeat in predictable ways?	• Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*

Numbers and Operations in Base Ten

Enduring Understandings In order to meet the standards, the students will need to understand that . . .	**Essential Questions** In order to understand, students will need to consider questions such as . . .	**Knowledge and Skills** Learning this material will require students to . . .
Generalize Place Value Understanding for Multi-Digit Whole Numbers • The base-ten numeration system is a scheme for recording numbers using the digits 0-9, groups of tens and place value. • Our number system is based on groups of ten. Whenever we get 10 in one place value, we move to the next greater place value. • In a multi-digit whole number, a digit is one place represents ten times what it would represent in the place immediately to its right. • Numbers can be read and written using base-ten numerals (i.e. 321), word form (i.e. three hundred twenty one) and in expanded form (3 x 100 + 2 x 10 + 1 x 1). • Place value can be used to compare and order numbers. The symbols < (less than), > (greater than), and = (equals) can be used to record comparisons in expressions. • Rounding whole numbers is a process for finding the multiple of 10, 100 and so on closest to a given number.	• How does a digit in any place value relate to the one to its right? • How are big numbers (up to 1,000,000) read and written in different forms? • How can whole numbers be compared and ordered? • How can whole numbers be rounded to any position?	• Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. • Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. • Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. • Use place value understanding to round multi-digit whole numbers to any place.

Enduring Understandings In order to meet the standards, the students will need to understand that . . .	**Essential Questions** In order to understand, students will need to consider questions such as . . .	**Knowledge and Skills** Learning this material will require students to . . .
Use Place Value Understanding and Properties of Operations to Perform Multi-Digit Multiplication • The standard addition and subtraction algorithms for multi-digit numbers break the calculation into simpler calculations using place value starting with the ones, then tens and so on. • For a given set of numbers, there are relationships that are always true called properties. Properties are the rules that govern arithmetic. (ie. The Commutative Property, the Associative Property and the Distributive Property of multiplication are three such properties.) • There is more than one algorithm for each of the operations with rational numbers. • Making an array with place-value blocks or area model provide ways to visualize and find products. • Basic facts and place value patterns can be used to find products when one factor is 10 or 100. • There is an expanded algorithm for multiplying where numbers are broken apart using place value and the parts are used to find partial products. The partial products are then added together to find the product.	• What are standard procedures for adding and subtracting multi-digit numbers? • How can arrays and area models be used to find products of multi-digit numbers? • What is a standard procedure for multiplying multi-digit numbers? • How can the inverse relationship between multiplication and division be helpful in knowing all multiplication and division facts through 12 x 12? • How can place value strategies, properties of operations and/or the relationship between multiplication and division be used to find whole-number quotients and remainders? • How can division situations be modeled using rectangular arrays and/or area models? • What is the standard procedure for dividing whole numbers?	• Fluently add and subtract multi-digit whole numbers using the standard algorithm. • Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. • *MA.5a.* Know multiplication facts and related division facts through 12 x 12. • Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Enduring Understandings In order to meet the standards, the students will need to understand that . . .	**Essential Questions** In order to understand, students will need to consider questions such as . . .	**Knowledge and Skills** Learning this material will require students to . . .
Multi-Digit Multiplication (cont.) • The standard multiplication algorithm is just a shortened way of recording the information in the expanded multiplication algorithm. Regrouping is used rather than showing all partial products. • The standard algorithm involves breaking apart numbers using place value, finding partial products, and then adding partial products to get the final product. The process is the same regardless of the size of the factors. • The inverse relationship between multiplication and division can be used to find all division facts. • Basic facts and place value patterns can be used to divide multiples of 10 or 100 by one-digit numbers. • Repeated subtraction situations can be solved using a division algorithm. • The sharing interpretation of division can be used to model the standard division algorithm. • The standard division algorithm breaks the calculation into simpler calculations using basic facts, place value, the relationship between multiplication and division, and estimation.		

Numbers and Operations – Fractions

(Grade 4 expectations are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12 and 100)

<table>
<tr><th>Enduring Understandings
In order to meet the standards, the students will need to understand that . . .</th><th>Essential Questions
In order to understand, students will need to consider questions such as . . .</th><th>Knowledge and Skills
Learning this material will require students to . . .</th></tr>
<tr><td>Extend Fraction Understanding and Equivalence
• Any number, measure, numerical expression, algebraic expression, or equation can be represented in an infinite number of ways that have the same value.
• The same fractional amount can be represented by an infinite set of different but equivalent fractions. Equivalent fractions are found by multiplying or dividing the numerator and denominator by the same nonzero number.
• Numbers, expressions, measures and objects can be compared and related to other numbers, expressions, measures and objects in different ways.
• Benchmark fractions such as ½ are useful when comparing two fractions to each other.
• Fraction models such as fraction bars and number lines are useful when determining fraction equivalence.
• Fractions can be built from unit fractions by applying and extending understandings of operations on whole numbers.</td><td>• How can the same fractional amounts be renamed using equivalent fractions?
• How can fractions be compared and ordered?
• How can benchmark fractions such as ½ be helpful when comparing fractions?
• What visual models are most useful when determining fraction equivalence?
• How can fractions be represented as the sum of unit fractions?
• What does it mean to add and subtract fractions and mixed numbers?
• What is a standard procedure for adding and subtracting fractions and mixed numbers with like denominators?
• How can fractions and mixed numbers be added and subtracted using visual models?
• How can visual models be used to help with multiplying a whole number by a fraction?
• How can understanding multiplication be helpful in multiplying fractions by whole numbers?</td><td>• Explain why a fraction a/b is equivalent to a fraction (n x a)/(n x b) by using visual fraction models.
• Compare two fractions with different numerators and different denominators by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions by using a visual fraction mode.
• Understand a fraction a/b with a > 1 as a sum of fractions 1/b.
• Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
• Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation.
• Add and subtract mixed numbers with like denominators.</td></tr>
</table>

Enduring Understandings In order to meet the standards, the students will need to understand that . . .	Essential Questions In order to understand, students will need to consider questions such as . . .	Knowledge and Skills Learning this material will require students to . . .
Fractions (cont.) • A model can be used to add or subtract two or more fractions. • When adding or subtracting fractions with like denominators, you are adding or subtracting portions of the same size. So, you can add or subtract the numerators without changing the denominators. • One way to add mixed numbers is to add the fractional parts and then add the whole number parts. Sometimes whole numbers or fractions need to be renamed. • One way to subtract mixed numbers is to subtract the fractional parts and then the whole number parts. Sometimes whole numbers or fractions need to be renamed. • Models can be used to show different ways of adding and subtracting mixed numbers. • Positive fractions can be added or subtracted by locating a fraction on the number line and then moving right to add or left to subtract. • Fractional amounts greater than 1 can be represented using a whole number and a fraction. Whole numbers can be represented as fractions. When the numerator and denominator are equal, the fraction equals 1. • A fractional amount can be decomposed into a sum of fractions in more than one way.		• Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators. • Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. • Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product 5 x (1/4), recording the conclusion by the equation 5/4 = 5 x (1/4). • Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. • Solve word problems involving multiplication of a fraction by a whole number by using visual fraction models and equations to represent the problem. • Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. • Use decimal notation for fractions with denominators 10 or 100. (i.e. Rewrite 0.62 as 62/100 ; describe a length as 0.62 meters; locate 0.62 on a number line diagram.)

Enduring Understandings In order to meet the standards, the students will need to understand that . . .	**Essential Questions** In order to understand, students will need to consider questions such as . . .	**Knowledge and Skills** Learning this material will require students to . . .
Fractions (cont.) • Physical representations and symbols can be used to develop the understanding that a/b = a x 1/b. • Models can be used to find the product of a whole number and a fraction. • To multiply a fraction by a whole number, one must multiply the whole number by the numerator of the fraction and then divide the product by the denominator of the fraction. • Understand decimal notation for fractions and compare decimal fractions. • Any number, measure, numerical expression, algebraic expression, or equation can be represented in an infinite number of ways that have the same value. • A decimal is another name for a fraction. • Each fraction, mixed number, and decimal can be associated with a unique point on the number line. • Decimal numeration is just an extension of whole number numeration. • Place value can be used to compare and order decimals. • Relationships among dollars, dimes and pennies are a good model for decimal numeration.		• Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions by using a visual model.

Measurement and Data

Enduring Understandings In order to meet the standards, the students will need to understand that . . .	**Essential Questions** In order to understand, students will need to consider questions such as . . .	**Knowledge and Skills** Learning this material will require students to . . .
Solve Problems Involving Measurement and Conversion of Measurement From a Larger Unit to a Smaller Unit • Some attributes of objects are measurable and can be quantified using unit amounts. • Length can be estimated and measured in different systems (customary, metric) and using different units in each system that are related to each other. • Capacity is a measure of the amount of liquid a container can hold. Capacity can be measured in different systems (i.e. customary, metric) and using different units that are related to each other. • Weight of an object is a measure of how heavy it is. • Mass is the measure of the quantity of matter in an object. Weight and Mass are different measures. • Time can be expressed using different units that are related to each other. • Relationships between customary measurement units can be expressed as a function (i.e. 12 inches to 1ft or 12 in. = 1 ft). • Relationships between metric measurement units can be expressed as a function (e.g. 10 mm to 1cm or 10 mm = 1cm). • Some problems can be solved by applying the formula for the perimeter of a rectangle or the formula for the area of a rectangle.	• What are customary and metric units for measuring length, capacity and weight/mass, and how are they related? • What do area and perimeter mean and how can each be found?	• Know relative sizes of measurement units within one system of units, including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. • Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. • Apply the area and perimeter formulas for rectangles in real-world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.

Enduring Understandings In order to meet the standards, the students will need to understand that . . .	**Essential Questions** In order to understand, students will need to consider questions such as . . .	**Knowledge and Skills** Learning this material will require students to . . .
Represent and Interpret Data • Some data can be represented using a line plot and the line plot can be used to answer certain questions about the data.	• How can line plots and other tools help to solve measurement problems?	• Make a line plot to display a data set of measurements in fractions of a unit (i.e. 1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots.
Geometric Measurement: Understand Concepts of Angle and Measure Angles • Line segments and rays are sets of points that describe parts of lines, shapes and solids. Angles are formed by two intersecting lines or by rays with a common endpoint and are classified by size. • The measure of an angle depends upon the fraction of the circle cut off by its rays. • The unit for measuring the size of the opening of an angle is 1 degree. • Angle measure can be added or subtracted.	• How can lines, angles and shapes be described, analyzed and classified? • How are angles measured, added and subtracted?	• Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement. • Know that an angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle" and can be used to measure angles. • Know that an angle that turns through n one-degree angles has an angle measure of n degrees. • Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. • Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts.

Geometry

Enduring Understandings In order to meet the standards, the students will need to understand that . . .	**Essential Questions** In order to understand, students will need to consider questions such as . . .	**Knowledge and Skills** Learning this material will require students to . . .
Draw and Identify Lines and Angles, and Classify Shapes by Properties of Their Lines and Angles • Point, line and place are the core attributes of space objects, and real-world situations can be used to think about these attributes. • Line segments and rays are sets of points that describe parts of lines, shapes and solids. Angles are formed by two intersecting lines or by rays with a common endpoint and are classified by size. • Two-dimensional or plane shapes have many properties that make them different from one another. Polygons can be described and classified by their sides and angles. • Some shapes can be reflected across one or more lines passing through the shape so the shape folds into itself. This is called the line of symmetry.	• How can lines, angles and shapes be described, analyzed and classified? • How can lines of symmetry be recognized and drawn in two-dimensional figures?	• Draw points, lines, line segments, rays, angles (i.e. right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures. • Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. • Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. • Identify line-symmetric figures and draw lines of symmetry.